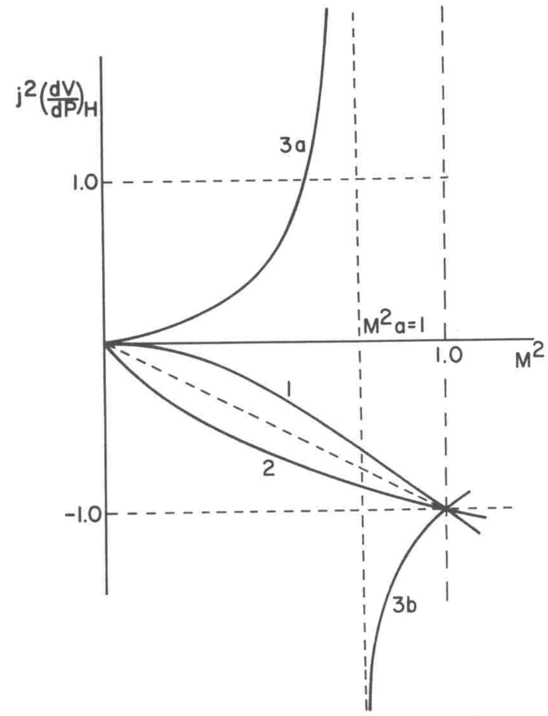

FIG. 1. Plot of $j^2(dV/dP)_H$ as function of M^2 for various values of a. Branch 1: $0<a<1$; Branch 2: $a<0$; Branch 3: $1<a$; Branch 3b: $M^2a>1$.

III. STABILITY WITH RESPECT TO TWO-DIMENSIONAL PERTURBATIONS

In this section we summarize the results of studies by D'yakov and by Erpenbeck of the structural stability of shocks with respect to two-dimensional perturbations.[2,3,5] These results are of special interest in the present context because the limits derived also correspond to the absolute instability limits for breakup of a plane shock into two waves, derived by Bethe.[1] This correspondence was first pointed out by Gardner.[4]

The results of these studies show that shock waves are unstable outside the limits given by

$$-1 \leq j^2(dV/dP)_H \leq 1+2M \ . \tag{17}$$

When either of these inequalities is exceeded, small sinusoidal perturbations of the front grow in amplitude with time.

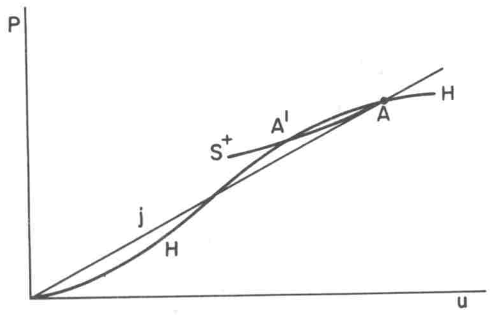

FIG. 2. Unstable Hugoniot curve, $j^2(dV/dP)_H<-1$. Hugoniot, H, and characteristic curve, S^+, lie above Rayleigh line, j, at A. Subsonic condition, $M<1$, violated at A.

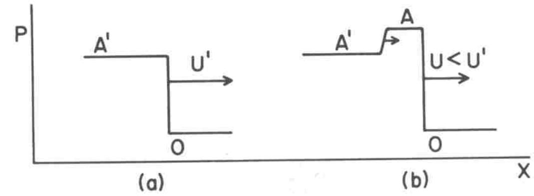

FIG. 3. Alternative wave solutions consistent with Hugoniot of Fig. 2.

It is remarkable that the limits of Ineq. (17) are also those for which a shock can split into two waves. That is, outside either limit a prescribed pressure, particle-velocity boundary condition can be satisfied by either a single shock or by a two-wave configuration. First, consider a case in which the lower limit is violated. Then, it is clear from Fig. 1 that, provided $M^2a<1$, the only solutions consistent with the jump conditions correspond to $M^2>1$. However, this implies that the shock travels faster than the speed of sound in the compressed medium behind the shock, and it has been shown that the Second Law would then be violated in the shock transition.[9] It will be shown later that the branch $M^2a>1$ is also unstable.

Another point of view that can be taken is illustrated in Fig. 2, which shows a Hugoniot curve in the P-u plane for which the lower limit of Ineq. (17) is violated at point A. The isentropic curve through point A intersects the Hugoniot curve again at point A′. We note that both the Hugoniot curve and the isentropic curve must lie on the same side of the Rayleigh line and are simultaneously tangent to that line at the lower stability limit of Ineq. (17). This is shown by Eq. (10), which can be inverted to give

$$j^2\left(\frac{dV}{dP}\right)_H = 1 - 2j\left(\frac{du}{dP}\right)_H \ ,$$

so that

$$-1 < j^2\left(\frac{dV}{dP}\right)_H$$

implies

$$j\left(\frac{du}{dP}\right)_H < 1 \ .$$

Moreover, as noted previously, when $M^2a<1$, this same restriction implies $M<1$, and from Eq. (13),

$$j\left(\frac{du}{dP}\right)_s < 1 \ .$$

This result has also been discussed by Landau and Lifshitz (Ref. 10, p. 326).

The configuration shown in Fig. 2 admits two solutions for prescribed boundary conditions corresponding to state A′. These are (a) a single shock to A′, and (b) a shock to state A followed by a slower rarefaction wave to A′, as illustrated in Fig. 3. In order for (b) to be a stable configuration (and a single shock to A to be unstable) the speed of the rarefaction wave must be less than the speed of the shock, i.e., the shock must be supersonic with respect to the medium behind, or $M>1$.

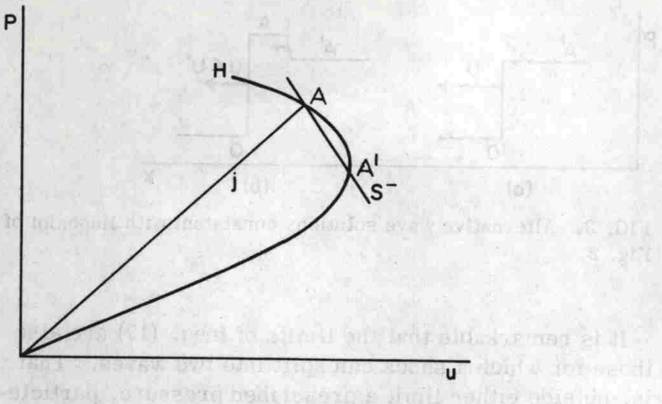

FIG. 4. Unstable Hugoniot curve, $j^2(dV/dP)_H > 1+2M$. Characteristic curve, S^-, intersects Hugoniot, H, twice, at A and A′.

An analogous argument applies when the upper limit of Ineq. (17) is violated. In this case, using Eq. (10),

$$j^2\left(\frac{dV}{dP}\right)_H = 1 - 2j\left(\frac{du}{dP}\right)_H > 1+2M\ ,$$

or, since $j>0$,

$$\left(\frac{du}{dP}\right)_H < -\frac{M}{j}\ .$$

Employing Eq. (13) this implies, for the negative solution of Eq. (13),

$$\left(\frac{dP}{du}\right)_s < \left(\frac{dP}{du}\right)_H < 0\ .$$

A configuration satisfying this inequality is shown in Fig. 4; the isentrope through state A crosses the Hugoniot curve again at state A′. A prescribed P-u state at the boundary corresponding to state A′ can then be satisfied by two different wave configurations: (a) a single shock to state A′, or (b) a shock to state A and a rarefaction to state A′ traveling in the opposite direction to the shock. These solutions are illustrated in Fig. 5.

It is thus clear that the limits of Ineq. (17) correspond to the limits outside which a shock can spontaneously split into two waves. These limits are illustrated in the P-V plane in Fig. 6.

It has been noted previously that the region for which

$$j(du/dP)_H < 0$$

is peculiar in that it admits multi-valued solutions to an impact problem.[5] Figure 7 shows an impedance-match solution in the P-u plane for a projectile with normal

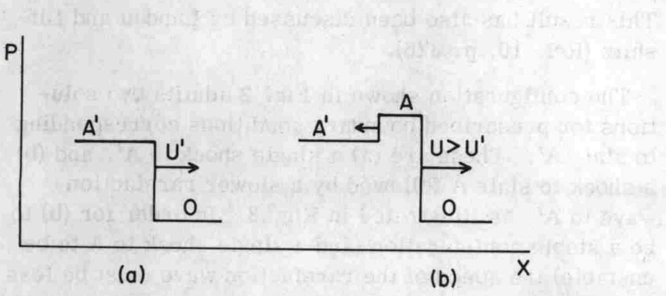

FIG. 5. Alternative wave solutions consistent with Hugoniot of Fig. 4.

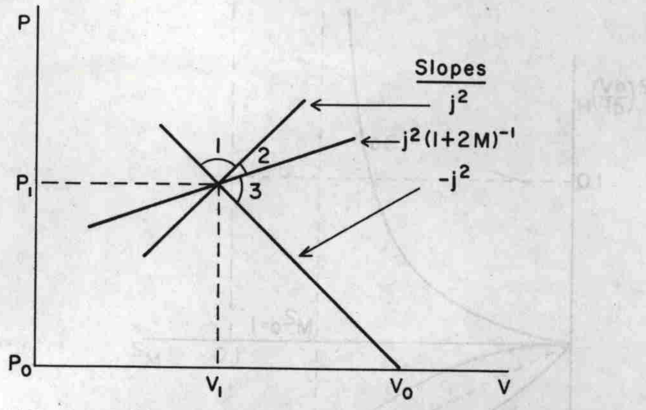

FIG. 6. Stable and unstable regions of P-V plane. Hugoniot curves with slopes in region 3 are unstable according to Eq. (17). In region 2, $j(du/dP)_H < 0$.

Hugoniot curve impacting a target whose Hugoniot curve does not violate Ineq. (17), but which contains a region in which $j(du/dP)_H < 0$. The two solutions for the common P-u state at the interface are indicated by A and B. This indeterminancy of the solution to an impact problem suggests that the criteria of Ineq. (17) are insufficient to guarantee stability. This possibility is examined further in the following sections.

IV. REFLECTION OF ACOUSTIC WAVES AT SHOCK FRONTS

Since a shock travels with subsonic velocity with respect to the compressed medium behind the shock, small amplitude, or acoustic waves in the compressed medium will overtake and reflect from the front. Figure 8(a) shows a diagram of such a reflection in the time-distance plane, and Fig. 8(b) is the corresponding diagram in the pressure-particle velocity plane. The Hugoniot curve is labeled H and the characteristic curves, Eq. (12b), by S+ and S− in the P-u plane. State

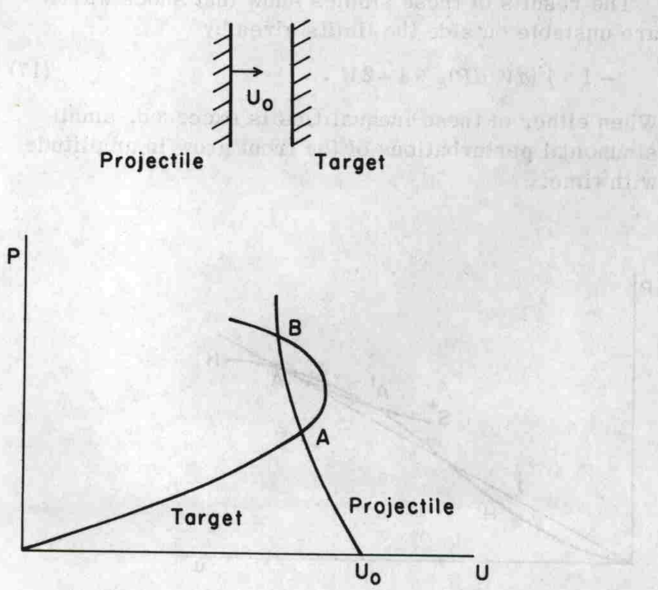

FIG. 7. Impedance match solution for impact of a projectile with a target whose Hugoniot contains a region for which $(dP/du)_H < 0$. States A and B satisfy interface conditions.